Useful Identities in Vector Notation

A. Dot Products

1. $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = \mathbf{v}\cdot(\mathbf{w}\times\mathbf{u})$
2. $\mathbf{uv}\cdot\mathbf{w} = \mathbf{w}\mathbf{v}\cdot\mathbf{u} + \mathbf{v}\times(\mathbf{u}\times\mathbf{w})$
3. $\mathbf{u}\cdot\nabla\mathbf{u} = (1/2)\nabla(\mathbf{u}\cdot\mathbf{u}) - \mathbf{u}\times(\nabla\times\mathbf{u})$
4. $\mathbf{u}\cdot\nabla s = \nabla\cdot(s\mathbf{u}) - s(\nabla\cdot\mathbf{u})$

B. Cross Products

1. $\mathbf{u}\times(\mathbf{v}\times\mathbf{w}) = \mathbf{v}\mathbf{u}\cdot\mathbf{w} - \mathbf{w}\mathbf{u}\cdot\mathbf{v}$
2. $\mathbf{u}\times(\nabla\times\mathbf{u}) = (1/2)\nabla(\mathbf{u}\cdot\mathbf{u}) - \mathbf{u}\cdot\nabla\mathbf{u}$

C. Divergence

1. $\nabla\cdot(s\mathbf{v}) = (\nabla s)\cdot\mathbf{v} + s(\nabla\cdot\mathbf{v})$
2. $\nabla\cdot(\mathbf{u}+\mathbf{v}) = \nabla\cdot\mathbf{u} + \nabla\cdot\mathbf{v}$
3. $\nabla\cdot(\mathbf{u}\times\mathbf{v}) = \mathbf{v}\cdot(\nabla\times\mathbf{u}) - \mathbf{u}\cdot(\nabla\times\mathbf{v})$
4. $\nabla\cdot\nabla\mathbf{u} = \nabla^2\mathbf{u} = \nabla(\nabla\cdot\mathbf{u}) - \nabla\times(\nabla\times\mathbf{u})$
5. $\nabla\cdot\nabla^2\mathbf{u} = \nabla^2(\nabla\cdot\mathbf{u})$
6. $\nabla\cdot(\nabla\times\mathbf{v}) = 0$
7. $\nabla\cdot(s\underline{\underline{\mathbf{T}}}) = (\nabla s)\cdot\underline{\underline{\mathbf{T}}} + s(\nabla\cdot\underline{\underline{\mathbf{T}}})$
8. $\nabla\cdot(\mathbf{uv}) = (\nabla\cdot\mathbf{u})\mathbf{v} + \mathbf{u}\cdot\nabla\mathbf{v}$
9. $\nabla\cdot(\underline{\underline{\mathbf{T}}}\cdot\mathbf{v}) = (\nabla\cdot\underline{\underline{\mathbf{T}}})\cdot\mathbf{v} + \underline{\underline{\mathbf{T}}}^t : \nabla\mathbf{v}$
10. $\nabla\cdot(\nabla\mathbf{v})^t = \nabla(\nabla\cdot\mathbf{v})$
11. $\nabla\cdot(\underline{\underline{\mathbf{I}}}s) = \nabla s$ (where $\underline{\underline{\mathbf{I}}}$ is the unit tensor)
12. $\nabla\cdot\mathbf{v} = \underline{\underline{\mathbf{I}}} : \nabla\mathbf{v}$

D. Gradient

1. $\nabla(\mathbf{u}+\mathbf{v}) = \nabla\mathbf{u} + \nabla\mathbf{v}$
2. $\nabla(\mathbf{u}\cdot\mathbf{v}) = \mathbf{u}\cdot\nabla\mathbf{v} + \mathbf{v}\cdot\nabla\mathbf{u} + \mathbf{u}\times(\nabla\times\mathbf{v}) + \mathbf{v}\times(\nabla\times\mathbf{u})$
3. $\nabla(st) = s\nabla t + t\nabla s$
4. $\nabla(\nabla\cdot\mathbf{v}) = \nabla\cdot(\nabla\mathbf{v})^t$

E. Curl

1. $\nabla\times(\mathbf{u}+\mathbf{v}) = \nabla\times\mathbf{u} + \nabla\times\mathbf{v}$
2. $\nabla\times(\nabla s) = \mathbf{0}$
3. $\nabla\times(s\mathbf{u}) = s(\nabla\times\mathbf{u}) + (\nabla s)\times\mathbf{u}$
4. $\nabla\times(\mathbf{u}\times\mathbf{v}) = \mathbf{u}(\nabla\cdot\mathbf{v}) - \mathbf{v}(\nabla\cdot\mathbf{u}) + \mathbf{v}\cdot\nabla\mathbf{u} - \mathbf{u}\cdot\nabla\mathbf{v}$
5. $\nabla\times(\nabla\times\mathbf{u}) = \nabla(\nabla\cdot\mathbf{u}) - \nabla^2\mathbf{u}$
6. $\nabla\times(\nabla^2\mathbf{u}) = \nabla^2(\nabla\times\mathbf{u})$
7. $\nabla\times(\mathbf{uv}) = (\nabla\times\mathbf{u})\mathbf{v} - \mathbf{u}\times(\nabla\mathbf{v})$

F. Laplacian

1. $\nabla^2\mathbf{u} = \nabla\cdot\nabla\mathbf{u} = \nabla(\nabla\cdot\mathbf{u}) - \nabla\times(\nabla\times\mathbf{u})$
2. $\nabla^2(\mathbf{u}+\mathbf{v}) = \nabla^2\mathbf{u} + \nabla^2\mathbf{v}$
3. $\nabla^2(\nabla\cdot\mathbf{u}) = \nabla\cdot(\nabla^2\mathbf{u})$
4. $\nabla^2(\nabla\times\mathbf{u}) = \nabla\times(\nabla^2\mathbf{u}) = -\nabla\times[\nabla\times(\nabla\times\mathbf{u})]$
5. $\nabla^2(st) = s(\nabla^2 t) + 2\nabla s\cdot\nabla t + t(\nabla^2 s)$
6. $\nabla^2(\nabla s) = \nabla(\nabla^2 s)$